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Finite tensor deformations of supergravity solitons

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Abstract: We consider brane solutions where the tensor degrees of freedom are excited. Explicit solutions to the full non-linear supergravity equations of motion are given for the M5 and D3 branes, corresponding to finite selfdual tensor or Born–Infeld field strengths. The solutions are BPS-saturated and half-supersymmetric. The resulting metric space-times are analysed.

Keywords: M-Theory, D-branes, Supergravity Models.

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Contents

1. Introduction

The way in which branes in M theory and string theory arise as "soliton" solutions of 11- or 10-dimensional supergravity is well known, see e.g. [1, 2]. Much less explored is the exact relation between the dynamics of the brane degrees of freedom and the target space fields. The former of course arise as zero-modes of the latter around a solitonic solution [3, 4], but when one goes beyond a linear approximation, no such relation has been established so far. Part of the motivation of the present work is to fill this gap. Specifically, we address the question of finding solutions to the supergravity equations of motion corresponding to finite excitations of the tensorial degrees of freedom, while keeping the brane flat and infinite. The analysis is applied to the M5 brane of 11-dimensional supergravity and the D3 brane of type II B supergravity, which both are truly solitonic. There are *a priori* strong reasons to believe that analytic solutions exist, since they are related to the dynamics of Born–Infeld vector fields and selfdual tensors on the world-volumes of the D3 and M5 branes, respectively. This calculation is carried through in section 2. Section 3 examines the metric properties of the resulting space-time, especially a limiting case for maximal field strength, where no asymptotic Minkowski region exists. In section 4, we show that the solutions are half-supersymmetric and construct the corresponding Killing spinors.

2. Finite tensor deformations

We want to find exact solutions for the M5 and D3 branes, where we have finite field strength deformations. What makes it possible to find exact solutions are the nice algebraic properties of the selfdual field strengths we are dealing with. For most of our conventions and notation we refer to ref. [4]. Here we just state our notation for the different types of indices occurring:

- Space-time indices: $M, N, \ldots$ (coordinate-frame), $A, B, \ldots$ (inertial);
- Longitudinal indices: $\mu, \nu, \ldots$ (coordinate-frame), $i, j, \ldots$ (inertial);
- Transverse indices: $p, q, \ldots$ (coordinate-frame), $p', q', \ldots$ (inertial).

2.1 The M5 brane

The 4-form field strength H should be parametrised by a closed 3-form $F(x)$ lying in the longitudinal directions, according to experience from brane dynamics [5, 6]. This can also be understood from the general Goldstone analysis [4]. In contrast to the (infinitesimal) Goldstone analysis, where F fulfilled a linear selfduality relation, F should in the exact analysis fulfill some non-linear selfduality relation. We are going to treat the simplest case where F is constant. Consider the equation of motion for H, $d{\star}H - \frac{1}{2}H \wedge H = 0$ [7]. The $\star$ operation involves the dualisation with the metric restricted to the 6-dimensional longitudinal directions. In the Goldstone analysis [4], where we considered an infinitesimal excitation of F, this metric was proportional to $\mathbb{1}$ and we did not have to care much about whether we had the radius-independent tensor in coordinate-frame or inertial indices, they just differed by a scalar function of the radial coordinate. Now, the dualisation in coordinate-frame indices involves a metric that will be "non-trivial", and for the selfduality to be consistent with radius-independence it must be possible to formulate it in terms of an inertial tensor.

Take h_{ijk} to be a (linearly) anti-selfdual inertial tensor. Define $q_{ij} = \frac{1}{2}h_i{}^{kl}h_{jkl}$. Then, $\operatorname{tr} q = 0$ and $q^2 = \mu\mathbb{1}$, where $\mu = \frac{1}{6}\operatorname{tr} q^2$. The tensor $(qh)_{ijk} \equiv q_i{}^l h_{ljk}$ is automatically antisymmetric and selfdual. For later purposes, we define $\nu = \frac{1}{2}\sqrt{\mu}$. The most general Ansatz for the deformed 4-form is now

$$
\begin{aligned}
H_{\mu\nu\lambda p} &= e_\mu{}^i e_\nu{}^j e_\lambda{}^k \partial_p \Delta F_{ijk}\,, \\
F_{ijk} &= f h_{ijk} + g(qh)_{ijk}\,,
\end{aligned}
\tag{2.1}
$$

where f and g are functions of μ and of the radial coordinate ρ. Due to the algebraic properties of h all higher order terms can be reduced to the two terms in the Ansatz. The necessity to include the second term is that the radial derivative on $\star H$ acts not only on the tensor but also on the vielbeins. The field along the 4-sphere will not change, since the magnetic charge should not be altered, so the background solution [8] remains unaltered

$$H_{pqrs} = \delta^{tu}\varepsilon_{pqrst}\partial_u\Delta\,, \tag{2.2}$$

where Δ is a harmonic function of the transverse coordinates, i.e., $\delta^{pq}\partial_p\partial_q\Delta = 0$. By considering all functions, as f and g above, as functions of Δ instead of ρ, one covers AdS space ($\Delta = (\frac{R}{\rho})^3$) as well as the asymptotically flat brane solutions ($\Delta = 1 + (\frac{R}{\rho})^3$), without any extra calculational complication.

As an Ansatz for the vielbeins, we take

$$\begin{aligned} e_\mu{}^i &= \delta_\mu{}^j\left(a\delta_j{}^i + bq_j{}^i\right)\,, \\ e_p{}^{p'} &= c\delta_p{}^{p'}\,, \end{aligned} \tag{2.3}$$

where a, b and c are functions of μ and Δ. One thing that makes the calculations simpler is that all matrices that may occur, vielbeins and derivatives of vielbeins, commute with each other. One may quite easily calculate the Ricci tensor. A first observation is that the RHS of Einstein's equations can never contain $\partial_p\partial_q\Delta$, so such terms must not be present in R_{pq}. This implies that $c = (\det e_\mu{}^i)^{-1/\tilde{d}}$, where $\tilde{d} = D - d - 2$[1]. When this is used, the Ricci tensor is, expressed in terms of $A \equiv \log e$ (e denoting $e_\mu{}^i$),

$$\begin{aligned} R_{pq} &= -\partial_p\Delta\partial_q\Delta\left(\mathrm{tr}(A'^2) + \frac{1}{\tilde{d}}(\mathrm{tr}A')^2\right) + \frac{1}{\tilde{d}}\delta_{pq}(\partial\Delta)^2\mathrm{tr}A''\,, \\ R_{\mu\nu} &= -c^{-2}(\partial\Delta)^2 e_\mu{}^i e_\nu{}^j A''_{ij}\,. \end{aligned} \tag{2.4}$$

Prime denotes differentiation w.r.t. Δ and $(\partial\Delta)^2 \equiv \delta^{pq}\partial_p\Delta\partial_q\Delta$. The matrix A will be parametrised as $A = \frac{1}{d}(\alpha\mathbb{1} + \beta q)$, and α is actually equal to $\log\det e_\mu{}^i$. It is also convenient to rescale the functions in the Ansatz for H as $\phi = e^{-\alpha}f$, $\psi = e^{-\alpha}g$. The remaining part of Einstein's equations, together with the e.o.m. for H, are now

$$\begin{aligned} 0 &= \alpha'' - e^{2\alpha}(1 - 2\mu\phi\psi)\,, \\ 0 &= \beta'' + 3e^{2\alpha}(\phi^2 + \mu\psi^2)\,, \\ 0 &= \alpha'^2 + \frac{1}{3}\mu\beta'^2 - e^{2\alpha}(1 - 4\mu\phi\psi)\,, \\ 0 &= \phi' + (e^\alpha + \frac{1}{2}\alpha')\phi - \frac{1}{2}\mu\beta'\psi\,, \\ 0 &= \psi' - (e^\alpha - \frac{1}{2}\alpha')\psi - \frac{1}{2}\beta'\phi\,. \end{aligned} \tag{2.5}$$

This is one equation too many, but by differentiating the third equation one gets a combination of the other four (eventually, one has to check that the integration

[1] D is the target space dimension and d that of the brane. Thus, in this case $\tilde{d}$=3.

constant vanishes). The μ-dependence can be removed by redefining $\mu^{1/4}\phi \to \phi$, $\mu^{3/4}\psi \to \psi$, $\mu^{1/2}\beta \to \beta$; the equations become identical to the ones above with $\mu = 1$.

The background solution, describing either $AdS_7 \times S^4$ or an M5 brane with no tensor excitations, is $\alpha = -\log\Delta$ and the rest zero. If one builds up the solution order by order in the perturbation, one first solves the zero-mode equation for ϕ giving $\phi = k\Delta^{-1/2}$. This linearised solution then backreacts on the geometry giving the lowest order perturbation to $\beta \sim \Delta^{-1}$. This non-diagonal metric then forces the tensor to contain the other duality component, $\psi \sim \Delta^{-3/2}$, which in turn enforces a diagonal modification to the vielbein, i.e. of α, of the order Δ^{-2}. And so it goes on. This becomes an expansion in negative powers of Δ and at the same time in the constant k, which just determines the normalisation of h_{ijk}. The μ-dependence is reinserted by choosing $\mu^{-1/4}k = 1$ (so that ϕ starts out with $\Delta^{-1/2}$), which makes the expansion look like

$$\begin{aligned}
\alpha &\sim -\log\Delta + \mu\Delta^{-2} + \mu^2\Delta^{-4} + \cdots \\
\beta &\sim \Delta^{-1} + \mu\Delta^{-3} + \mu^2\Delta^{-5} + \cdots \\
\phi &\sim \Delta^{-1/2} + \mu\Delta^{-5/2} + \mu^2\Delta^{-9/2} + \cdots \\
\psi &\sim \Delta^{-3/2} + \mu\Delta^{-7/2} + \mu^2\Delta^{-11/2} + \cdots
\end{aligned} \tag{2.6}$$

Considering the first few terms in this expansion enabled us to find the exact solution:

$$\begin{aligned}
\alpha &= -\frac{1}{2}\log(\Delta^2 - \nu^2)\,, & \beta &= \frac{3}{4\nu}\log\frac{\Delta-\nu}{\Delta+\nu}\,, \\
\phi &= \frac{1}{2}\left(\frac{1}{\sqrt{\Delta+\nu}} + \frac{1}{\sqrt{\Delta-\nu}}\right), & \psi &= \frac{1}{4\nu}\left(\frac{1}{\sqrt{\Delta+\nu}} - \frac{1}{\sqrt{\Delta-\nu}}\right).
\end{aligned} \tag{2.7}$$

Before inserting the explicit solution for α and β in the metric, it is useful to note that the eigenvalues of the matrix q are $\pm 2\nu$, and that there are three of each. We group the longitudinal coordinates accordingly into $x_\pm$. The time direction is included in x_-. The metric then becomes

$$ds^2 = (\Delta^2-\nu^2)^{-1/6}\left[\left(\frac{\Delta+\nu}{\Delta-\nu}\right)^{1/2} dx_-^2 + \left(\frac{\Delta-\nu}{\Delta+\nu}\right)^{1/2} dx_+^2\right] + (\Delta^2-\nu^2)^{1/3}dy^2\,. \tag{2.8}$$

It clearly reduces to the well known M5 brane metric when the tensor deformation is absent, i.e., when $\nu = 0$. We will return to the properties of the metric in section 3. Finally, inserting the solution into the Ansatz (2.1) gives us the 4-form in inertial indices:

$$H_{ijkp'} = \frac{\delta_{p'}{}^p\partial_p\Delta}{(\Delta^2-\nu^2)^{2/3}}\left[\frac{1}{\sqrt{\Delta+\nu}}\Pi_+h + \frac{1}{\sqrt{\Delta-\nu}}\Pi_-h\right]_{ijk}\,, \tag{2.9}$$

where $\Pi_\pm = \frac{1}{2}(\mathbb{1} \pm \frac{q}{2\nu})$ project all indices on the $+$ and $-$ directions (the algebraic properties of h tell us that only $h_{i_+j_+k_+}$ and $h_{i_-j_-k_-}$ are non-vanishing).

2.2 The D3 brane

The relevant tensor field in type II B supergravity [9] is the complex 3-form field strength H. The D3 brane is invariant under SL(2;$\mathbb{Z}$) transformations, and it is convenient to keep SL(2;$\mathbb{Z}$) covariance throughout the calculations. The Bianchi identity and equation of motion for H are

$$\begin{aligned} DH - P\wedge\bar{H} &= 0\,,\\ D{\star}H - P\wedge{\star}\bar{H} + iG\wedge H &= 0\,, \end{aligned} \tag{2.10}$$

where the U(1) covariant derivative D contains a connection Q, which together with P are the left-invariant SL(2;$\mathbb{R}$) Maurer–Cartan forms built from the scalars.[2]

We use an Ansatz analogous to the M5 brane case:

$$H = d\Delta\wedge\tilde{F}\,, \tag{2.11}$$

where

$$\tilde{F}_{ij} = fF_{ij} + g\bar{F}_{ij}\,. \tag{2.12}$$

We again have one anti-selfdual (${\star}F = -iF$) and one selfdual (${\star}\bar{F} = i\bar{F}$) part. The algebraic properties of the matrix F are

$$\begin{aligned} (FF)_{ij} &= \mu\delta_{ij}\,, \qquad \mu = \frac{1}{4}\mathrm{tr}F^2\,,\\ (F\bar{F})_{ij} &= (F\bar{F})_{ji}\,,\\ \mathrm{tr}(F\bar{F}) &= 0\,. \end{aligned} \tag{2.13}$$

When we excite H we must also excite the 1-form P, as a consequence of the equations of motion, and we need an Ansatz for that too,

$$P = ud\Delta\,, \tag{2.14}$$

where $u = u(\mu,\bar{\mu},\Delta)$. The Bianchi identity and equation of motion for P are

$$\begin{aligned} DP &= 0\,,\\ D{\star}P - H\wedge{\star}H &= 0\,. \end{aligned} \tag{2.15}$$

The Ansätze trivially fulfill the Bianchi identity parts of (2.10) and (2.15) when only functions of the radial coordinate are considered.

The Ansatz for the vielbeins are

$$\begin{aligned} e_\mu{}^i &= \delta_\mu{}^j(a\delta_j{}^i + b(F\bar{F})_j{}^i)\,,\\ e_p{}^{p'} &= c\delta_p{}^{p'}\,, \end{aligned} \tag{2.16}$$

which is also completely analogous to the M5 brane case, i.e., $e_\mu{}^i$ is made up of the two symmetric matrices we can construct.

[2]We will leave Q out of the continued discussion — to the initiated reader it will be obvious that it is pure gauge, and we use this to put it to zero.

The Ricci tensor is given by eq. (2.4), where A is now parametrised as $A = \frac{1}{d}(\alpha \mathbb{1} + \beta F\bar{F})$. The equations of motion we want to solve are Einstein's equation

$$R_{MN} = 2\bar{P}_{(M}P_{N)} + \bar{H}_{(M}{}^{RS}H_{N)RS} - \frac{1}{12}g_{MN}\bar{H}_{RST}H^{RST} + \frac{1}{96}G_{(M}{}^{RSTU}G_{N)RSTU}\,,$$

together with the the equations of motion in (2.10) and (2.15). We use the background solution [10]

$$\begin{aligned} G = \pm\frac{1}{5!}\Big(&\delta^{mn}\partial_m\Delta\varepsilon_{npqrst}dy^p\wedge dy^q\wedge dy^r\wedge dy^s\wedge dy^t \\ &- 5g^{-2}\partial_m\Delta\varepsilon_{\mu\nu\rho\sigma}dy^m\wedge dx^\mu\wedge dx^\nu\wedge dx^\rho\wedge dx^\sigma\Big)\,, \end{aligned} \tag{2.17}$$

where $g = \det(g_{MN})$ (the first term, which gives the D3 brane charge, is identical to the one in the ordinary D3 brane solution, and the second is its dual, where we have taken into account that the metric is modified). With the same rescalings as for the M5 brane, i.e., $f = e^\alpha\phi$, $g = e^\alpha\psi$ and $u = e^\alpha\chi$, we can rewrite the equations of motion as

$$\begin{aligned} 0 &= \alpha'' - e^{2\alpha}\left(1 + 4\left(\mu\phi\bar{\psi} + \bar{\mu}\bar{\phi}\psi\right)\right)\,, \\ 0 &= \beta'' - 8e^{2\alpha}\left(\phi\bar{\phi} + \psi\bar{\psi}\right)\,, \\ 0 &= \alpha'^2 + \frac{1}{2}\mu\bar{\mu}\beta'^2 - e^{2\alpha}\left(1 + 8\left(\mu\phi\bar{\psi} + \bar{\mu}\bar{\phi}\psi\right) - 4\chi\bar{\chi}\right)\,, \\ 0 &= \phi' + \left(e^\alpha + \frac{1}{2}\alpha'\right)\phi - \frac{1}{2}\bar{\mu}\beta'\psi + e^\alpha\chi\bar{\psi}\,, \\ 0 &= \psi' - \left(e^\alpha - \frac{1}{2}\alpha'\right)\psi - \frac{1}{2}\mu\beta'\phi + e^\alpha\chi\bar{\phi}\,, \\ 0 &= \chi' + \alpha'\chi + 2e^\alpha\left(\mu\phi^2 + \bar{\mu}\psi^2\right)\,. \end{aligned} \tag{2.18}$$

By differentiating the third equation we get a combination of the other five. The first three equations come from Einstein's equation, the fourth and fifth from the equation for H and the last one from the equation for P. From the properties of the fields involved under U(1) gauge transformations it is clear that α, β and ϕ are real functions, while ψ and χ must be real functions multiplied by μ. The solution to the equations is given by

$$\begin{aligned} \alpha &= -\frac{1}{2}\log(\Delta^2 - \nu^2)\,, \\ \beta &= -\frac{2}{\nu}\log\frac{\Delta - \nu}{\Delta + \nu}\,, \\ \phi &= \frac{1}{2}\left(\frac{1}{\sqrt{\Delta + \nu}} + \frac{1}{\sqrt{\Delta - \nu}}\right)\,, \\ \psi &= -\frac{\mu}{\nu}\left(\frac{1}{\sqrt{\Delta + \nu}} - \frac{1}{\sqrt{\Delta - \nu}}\right)\,, \\ \chi &= \frac{\mu}{\sqrt{\Delta^2 - \nu^2}}\,, \end{aligned} \tag{2.19}$$

where $\nu = 2|\mu|$ and we have used the normalisation that $\phi \to \Delta^{-1/2}$ as $\mu \to 0$ (the same rescaling argument holds here as for the M5 brane).

The metric may be diagonalised in the same manner as the M5 brane metric (the eigenvalues of $F\bar{F}$ are $\pm\frac{\nu}{2}$, and now time is in the positive eigenvalue sector), giving

$$ds^2 = (\Delta^2 - \nu^2)^{-1/4} \left[\left(\frac{\Delta + \nu}{\Delta - \nu} \right)^{1/2} dx_+^2 + \left(\frac{\Delta - \nu}{\Delta + \nu} \right)^{1/2} dx_-^2 \right] + (\Delta^2 - \nu^2)^{1/4} dy^2 \,. \tag{2.20}$$

Inserting the solution into the Ansätze (2.11) and (2.14) finally gives us

$$\begin{aligned} H_{p'ij} &= \frac{\delta_{p'}{}^p \partial_p \Delta}{2(\Delta^2 - \nu^2)^{5/8}} \left[\frac{1}{\sqrt{\Delta + \nu}} \left(F - \frac{2\mu}{\nu} \bar{F} \right) + \frac{1}{\sqrt{\Delta - \nu}} \left(F + \frac{2\mu}{\nu} \bar{F} \right) \right]_{ij} \,, \\ P_{p'} &= \frac{\mu \delta_{p'}{}^p \partial_p \Delta}{(\Delta^2 - \nu^2)^{9/8}} \end{aligned} \tag{2.21}$$

in inertial indices.

We want to stress that the structures of the solutions for the D3 and M5 branes are completely analogous (except that we happen to excite additional scalar fields in the D3 brane case, which however is easily dealt with). The linear terms in the deformations, i.e., the lowest order terms in the series expansions of ϕ, agree with the zero-modes derived in ref. [4].

3. Properties of the metrics

The metric space-times described by eqs. (2.8) and (2.20) represent deformations of the original AdS$\times$sphere or brane space-times parametrised by one real number ν, measuring the square of the field strength. When the radial coordinate ρ runs from 0 (which is the horizon in the brane case and a subset of no special significance in the AdS case) to ∞, Δ runs from ∞ to 1 for the brane and from ∞ to zero for AdS. We see that there is potential danger when $\Delta - \nu$ becomes negative.

Let us first treat the AdS case. Here $\Delta - \nu = (\frac{R}{\rho})^{\tilde{d}} - \nu$, and this is bound to change sign at some finite radius when $\nu > 0$. The question is whether this is a physical singularity or not. It is straightforward to calculate e.g. the curvature scalar, and find that it diverges at this radius. Such solutions do not define sensible space-times.

For the brane solutions, $\Delta - \nu = (\frac{R}{\rho})^{\tilde{d}} + 1 - \nu$. The solution makes sense for $\nu \leq 1$. This is a reflection of the Born–Infeld or Born–Infeld-like dynamics, which breaks down at field strengths where $\det(g + F)$ vanishes. The behaviour of the solutions for small radii is always unmodified, i.e., AdS$_{d+1} \times S^{\tilde{d}+1}$. For large radii, there is an asymptotic Minkowski region as long as ν is strictly smaller than 1.

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The limiting case, $\nu = 1$, has some interesting properties. One may calculate the curvature scalar, and find that it is non-singular as $\rho \to \infty$; it goes asymptotically as ρ^{-1}. After some trivial rescalings, the leading terms in the metric behave as

$$\begin{aligned} \text{M5:} \qquad & ds^2 = \rho^2 dx_-^2 + \rho^{-1}(dx_+^2 + dy^2)\,, \\ \text{D3:} \qquad & ds^2 = \rho^3 dx_+^2 + \rho^{-1}(dx_-^2 + dy^2)\,. \end{aligned} \tag{3.1}$$

As $\rho \to \infty$, half of the longitudinal directions "expand" and the other half "shrink", and what remains is something rather like a continuously smeared membrane or string, respectively. Whether this interpretation is physically relevant is unclear to us, however it is supported by the asymptotic behaviour of the dual of the tensor field, which asymptotically lies in the shrinking directions and the $(\tilde{d} + 1)$-sphere. The limiting metric does not factorise, but it has some things in common with the AdS metric: the space-like distance to $\rho = \infty$ is infinite, but light rays may reach infinity (and come back) in finite time.

4. Supersymmetric properties of the solutions

In the absence of an expectation value for the field strength on the brane, it is well known that the solutions break half the supersymmetry, i.e., that there are 16 Killing spinors. Arguing naïvely in terms of the field theory on the brane, one might expect that giving a background value to F would break the entire remaining global supersymmetry, so that the solutions presented here would be non-supersymmetric (and perhaps less interesting). What actually happens is instead that there are new combinations of the broken and unbroken supersymmetries that become Killing spinors in the presence of $F \neq 0$, and that the new solutions enjoy the same amount of supersymmetry, 16 Killing spinors.

There are at least two good arguments why this should happen. The first, more conceptual, is that the tensor modes are very much on the same footing as the scalar ones, in the sense that they all result from breaking of large gauge transformations [4]. Deforming a brane by giving constant "field strength" to scalars (transverse coordinates) corresponds to tilting the brane through some angle, a somewhat trivial operation that of course does not change the number of supersymmetries. The definition of world-volume chirality however changes, and one has to recombine broken and unbroken supersymmetries to recover the new Killing spinors. A similar phenomenon should occur for the tensors, and we already know that an analogous mechanism is at work for the tensors themselves, where chirality (selfduality) becomes nonlinear. The second, more technical, argument is that one knows from work on κ-symmetry in supersymmetric brane dynamics [11, 5, 12, 6] that there is a half-rank projection matrix, or generalised chirality operator [13], acting on spinors separating broken and unbroken supersymmetry, and that this matrix generically depends on F. For constant F, this means that there should be 16 global supersymmetries.

When the tensor degrees of freedom are turned on, the branes carry not only magnetic charge, but also local electric charge [14, 4]. The BPS property expressed through the existence of a local projection on the Killing spinors involves both charges, which explains why the excited brane may be BPS-saturated although the tensor excitations carry energy. The configurations carrying global electric charge are world-volume solitons [15].

The most convincing argument is of course to construct the Killing spinors explicitly, which we now proceed to do (although we satisfy ourselves with the M5 brane case). The preserved supersymmetry obeys the Killing spinor equation obtained by setting the variation of the gravitino field in the background to zero:

$$\delta_\zeta \psi_M = D_M \zeta - \frac{1}{288}\left(\Gamma_M{}^{NPQR} - 8\delta_M{}^N \Gamma^{PQR}\right)\zeta H_{NPQR} = 0\,. \tag{4.1}$$

The inertial gamma matrices are split as $\Gamma_A = (\gamma_i, \gamma_7 \oplus \Sigma_{p'})$ The calculation is straightforward (along the lines of ref. [4]). After assuming that the only functional dependence comes through Δ, one obtains a differential equation for ζ,

$$\zeta' + \left[\frac{1}{3}\Delta(\Delta^2 - \nu^2)^{-1} + \frac{1}{4}(\Delta^2 - \nu^2)^{-1/2}\gamma_7\right]\zeta = 0\,, \tag{4.2}$$

and an algebraic condition

$$\frac{1}{2}(\mathbb{1} + \Gamma)\zeta = 0\,; \qquad \Gamma = \Delta^{-1}(\Delta^2 - \nu^2)^{1/2}\left(\gamma_7 + \frac{1}{12}(\Delta^2 - \nu^2)^{1/2} F_{ijk}\gamma^{ijk}\right). \tag{4.3}$$

It is now crucial that the last equation projects ζ on half the original number of components. Using the explicit forms of the functions entering into F gives $\Gamma^2 = \mathbb{1}$, so that eq. (4.3) is a projection. It defines a generalised chirality condition, which for any fixed radius takes the form known from the κ-symmetric formulation of the M5 brane [6]. The chirality condition varies continuously with the radial coordinate, as does the non-linear selfduality condition on F.

The solution to eq. (4.2) is

$$\begin{aligned}
\zeta_- &= (\Delta^2 - \nu^2)^{1/12}\left(\frac{1}{\sqrt{\Delta + \nu}} + \frac{1}{\sqrt{\Delta - \nu}}\right)^{1/2}\lambda_-\,, \\
\zeta_+ &= (\Delta^2 - \nu^2)^{-5/12}\left(\frac{1}{\sqrt{\Delta + \nu}} + \frac{1}{\sqrt{\Delta - \nu}}\right)^{-1/2}\lambda_+\,,
\end{aligned} \tag{4.4}$$

where ζ has been split in chirality components according to the eigenvalue of γ_7 and where $\lambda_\pm$ do not depend on Δ. We notice that in the absence of a tensor field we recover the Killing spinors of ref. [4] which was $\zeta = \Delta^{-1/12}\lambda_-$. It remains to be checked that the solutions (4.4) are consistent with the chirality (4.3), i.e., that the Δ-dependence cancels upon inserting the solutions into the chirality condition. This indeed happens, and the chirality condition condenses into

$$\lambda_+ = -\frac{1}{12}h_{ijk}\gamma^{ijk}\lambda_-\,, \tag{4.5}$$

which together with eq. (4.4) gives the explicit form of the Killing spinors.

5. Discussion

We have derived a new class of half-supersymmetric solutions of 11-dimensional and type II B supergravity, corresponding to M5 and D3 branes with non-vanishing constant field strength. The structure of the solutions clearly reflects the property of Born–Infeld-like dynamics as opposed to quadratic actions, in that there is a maximal allowed value of the field strength.

It is interesting to note that although the symmetry of the solutions is smaller than in the case of vanishing field strength—the longitudinal SO(1,5) part of the isometry group is broken into SO(1,2)×SO(3) for the M5 brane (and accordingly for the D3 brane), the amount of supersymmetry is unchanged (the longitudinal translations of course remain unbroken). The split of the longitudinal directions in two groups is a novel property of brane solutions. It is not related to the longitudinal symmetry breaking induced by world-volume solitons, rather this split seems to have something to do with other branes, in these cases membranes and strings. The phenomenon might deserve further study, especially in the strong field limit. The formalism of ref. [16] may be useful in this context.

It should be possible to push the analysis further by considering also configurations with field strengths that depend on the longitudinal coordinates and thus derive the dynamics of the fields (the result would be in the selfdual form of refs. [12, 6]). Another application would be the generalisation to other types of branes—the method presented here might provide a manifestly SL(2;$\mathbb{Z}$)-covariant formulation of the type II B 5-branes. Finally, it would be interesting to understand whether the limiting solutions of maximal field strength have some physical significance, considering their interesting asymptotic structure.

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